

1. Current Grammar. Again, we want ultimately to develop a system of rules that generate and interpret *all and only* sentences of English. We will constantly be revising our grammar to account for more and more sentences.

Our grammar has four components:

(i) *Inventory of denotations*

Let D be the set of all individuals that exist in the real world. Possible denotations are:

- Elements of D, the set of actual individuals.
- Elements of $\{0,1\}$, the set of truth-values.
- Subsets of D.
- Operations on sets: Intersection (\cap), Union (\cup), Complement ($'$).

(ii) *Lexicon*

- N: $[[Emery]]^s = \text{Emery}$, $[[Blendia]]^s = \text{Blendia}$, ...
- V: $[[smile]]^s = \{x \mid \text{smile}(x)(s)\}$, $[[laugh]]^s = \{x \mid \text{laugh}(x)(s)\}$
 be_1 (vacuous)
- T: be_2 (We will neglect for now the semantic contribution of the T node.)
- A: $[[nice]]^s = \{x \mid \text{nice}(x)(s)\}$, $[[weird]]^s = \{x \mid \text{weird}(x)(s)\}$, ...
- N: $[[student]]^s = \{x \mid \text{student}(x)(s)\}$, $[[cat]]^s = \{x \mid \text{cat}(x)(s)\}$...
- D: a (vacuous)
- Conj: $[[and]]^v = \cap$, $[[or]]^v = \cup$
- Neg: $[[not]]^v = '$

(iii) *Syntactic rules*

- S \rightarrow NP (T) VP
- NP \rightarrow (D) N VP \rightarrow V (NP) (AP) AP \rightarrow A
- VP \rightarrow Neg VP VP \rightarrow VP Conj VP

(iv) *Semantic rules of composition*

For any situation s ,

- (a) If α has the form $[_S \text{ NP (T) VP}]$, $[[\alpha]]^s = 1$ iff $[[\text{NP}]]^s \in [[\text{VP}]]^s$.
- (b) If α is a non-branching node whose daughter node is β , then $[[\alpha]]^s = [[\beta]]^s$.
- (c) If α is a terminal node, then $[[\alpha]]^s$ is specified in the lexicon.
- (d) If α has the form $[_{VP1} \text{ VP}_2 \text{ Conj VP}_3]$, $[[\alpha]]^s = [[\text{VP}_2]]^s [[\text{Conj}]]^s [[\text{VP}_3]]^s$.
- (e) If α has the form $[_{VP1} \text{ Neg VP}_2]$, $[[\alpha]]^s = [[\text{VP}_2]]^s [[\text{Neg}]]^s$.

Again, on what we've gained: One of our basic semantic abilities is that of matching sentences with situations. Our grammar can be seen as a way of representing our capacity to pair sentences with the situations that they describe. Sentence content can be regarded as a relation between sentences and situations, or circumstances. Our notation $[[S]]^s = 1$ (or 0) can be interpreted as saying that S correctly characterizes or describes (or does not correctly describe) situation s .

2. Practice

- (1) Kaline is a cat.
- (2) Kali is not listening.
- (3) Kali is not weird.

3. More Empirical Coverage

- (4) Jose is around.
- (5) Jose is not around.
- (6) Jolie is not a student.
- (7) Kali is happy or weird.
- (8) Kali is happy and not weird.
- (9) Nick is in and out.

4. Semantically Vacuous Words.

Certain lexical items are commonly assumed to make no semantic contribution to the structure in which they occur.

One case is the copula *be* in predicative sentences such as *Christian is ecstatic* and *Christian is around*. We would want the following equalities, for example:

$$(10) \llbracket is\ ecstatic \rrbracket^s = \llbracket ecstatic \rrbracket^s \quad (11) \quad \llbracket is\ around \rrbracket^s = \llbracket around \rrbracket^s$$

Another case is the indefinite article *a* when it occurs in predicate nominals such as:

$$(12) \quad \text{Christian is a student.} \quad (13) \quad \text{Kaline is a cat.}$$

Here too we want the following equalities:

$$(14) \llbracket a\ student \rrbracket^s = \llbracket student \rrbracket^s \quad (15) \quad \llbracket a\ cat \rrbracket^s = \llbracket cat \rrbracket^s$$

We will assume then that the semantic component simply “doesn’t see” the copula *be* or the indefinite article *a*. As a result, a structure that is binary branching may be treated as non-branching by the semantic rules, in that a branch occupied by a vacuous item doesn’t count.

5. Transitives: Ordered pairs, Cartesian products, and Relations

The meanings of intransitive verbs can be modeled by sets.

What about transitive verbs?

- (16) Karen saved David.
- (17) David appreciates Karen.
- (18) Blendia invited Carlos.

Intuitively, *save* denotes a relationship between two individuals, such that the first saved the second. In other words, we can think of *saved* as representing pairs of individuals, such that the first individual in the pair saved the second.

To capture this formally, we introduce the notion of an *ordered pair*.

If we have two objects *a* and *b*, we can construct from them the *ordered pair* $\langle a, b \rangle$, in which *a* is the first member of the pair, and *b* is the second. Here, order is crucial:

$$(19) \quad \langle a, b \rangle = \langle b, a \rangle \text{ iff } a = b.$$

Recall that members of a set, in contrast, are unordered: $\{a, b\} = \{b, a\}$.¹

In set theory, a *two-place relation* is a set of ordered pairs. We can represent the meaning of *save*, then, as a two-place relation. In particular, for any situation *s*, the denotation of *save* in *s* is the set of ordered pairs such that the first member of each pair saves the second in *s*:

$$(20) \quad \text{For any } s, \llbracket save \rrbracket^s = \{ \langle x, y \rangle \mid save(x)(y)(s) \}.$$

We can form ordered pairs out of any two sets *A* and *B* by taking an element of *A* as the first member of the pair and an element of *B* as the second member. The *Cartesian product* of *A* and *B*, written $A \times B$ is the set consisting of all ordered pairs whose first member belongs to *A* and whose second member belongs to *B*.

$$(21) \quad A \times B =_{\text{def}} \{ \langle x, y \rangle \mid x \in A \text{ and } y \in B \}$$

Practice: Assume the sets $A = \{Saturn, Jupiter\}$ and $B = \{me, you\}$.

What is:

- (a) $A \times B$
- (b) $B \times A$
- (c) $B \times B$

We write $R \subseteq A \times B$ for a relation between objects from two sets *A* and *B*, which we call a relation *from A to B*.

Thus, *save* denotes a relation from *D* to *D*, and $\llbracket save \rrbracket^s \subseteq D \times D$.

¹ Ordered pairs are set theoretic objects, as they can be defined using sets, as in (i); for our purposes, however, you don’t need to know this definition.

$$(i) \quad \langle a, b \rangle =_{\text{def}} \{ \{a\}, \{a, b\} \}$$