

0. From Last Time

Explain why *most* implicates *not all* in (1), but not in (2).

- (1) You answered most questions correctly.
- (2) Anyone who answered most questions correctly passed.

1. Where We're Going

Our goal

A theory of semantic competence.

Observations so far

A native speaker has intuitions about the kinds of inference relations that hold between sentences. He/she knows (at least tacitly) whether an utterance entails or implicates another.

Sentence meaning:

How can we represent what native speakers know about sentence meaning?

Proposal we will explore

To know the meaning of a sentence is to know its *truth conditions*.

Question

How do we understand sentences we have never heard before?

Principle of Compositionality (Frege)

The meaning of a sentence is computed from the meanings of its parts, and the way those parts are assembled syntactically.

Immediate goal

Explore how the meaning of a sentence is computed from the meanings of its parts.

How to reach that goal

Posit suitable set theoretic entities as meanings for sentence parts.

2. Truth-conditions and compositionality

Proposal: To know the meaning of a sentence is to know its truth conditions.

Consider:

- (3) Emery is sitting.

You may not know whether (3) is true, but you do know what the world would have to be like for it to be true. In other words, in knowing what (1) means, you have the ability to determine, given a possible situation, whether or not (3) is true or false in that situation. Based on this ability, we can say that knowing the meaning of a sentence involves *at least* knowing the conditions under which a sentence is true.

We want a theory of sentence meaning, then, that pairs sentences with their truth conditions. We want a theory that gives us, for any situation s , and any sentence S :

- (4) S is true in s iff p .

Where p describes the conditions that must obtain for S to be true in s .

What are situations? Situations describe the ways things are, have been, or could be. For example: the situation of us in this classroom right now, the situation of you waking up this morning, the situation of you on this date five years from now, etc.

Because there are an infinite number of sentences in English, we cannot just have memorized the truth conditions for every sentence. Rather, we must have some way of deriving the truth conditions of a sentence based on the semantic contributions of its parts, and the way in which they are assembled syntactically.

Principle of Compositionality

The meaning of a sentence is computed from the meanings of its parts, and the way those parts are assembled syntactically.

But what are the parts of a sentence, and how are those parts combined? And what does each part mean?

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3. Set Theory

A *set* is a collection of objects of any kind.

E.g., the set of students in this class, the set of letters in the English alphabet.

Objects in a set are called the *members* or *elements* of that set.

E.g., Evan is a member of the set of students in this class.

A set can be formed out of elements that have no connection whatsoever.

E.g., the set consisting of you, the word *me*, and the square root of 7.

Sets may have any number of elements, finite or infinite.

E.g., the set of socks in this room, the set of sentences of English.

A set with only one member is called a *singleton* set.

E.g., the set with only you in it.

Sets themselves may be members of other sets.

E.g., the set of all sets consisting of you and a planet.

There is one set with no members at all, the *empty set* or *null set*, symbolized as \emptyset .

Notational conventions. We will use capital letters A, B, C, \dots for sets and lower-case letters a, b, c, \dots or sometimes x, y, z, \dots for the members of sets. We will use \in to symbolize the membership relation, so that $b \in A$ is read as ' b is a member of A '. If we want to say that b is not a member of A , we write $b \notin A$.

3.1 Specification of sets

There are at least two ways to specify a set:

List notation. To specify a set by list notation, the names of the members, separated by commas, are enclosed in braces. E.g., {Kali, Joel, Karen}.

Predicate notation. To specify a set by *predicate notation*, we specify a property that the members of the set share.

E.g., $\{x \mid x \text{ is a sentence of Tagalog}\}$,
 $\{x \mid x \text{ is a student in this class whose first name starts with } K\}$.

The vertical line after the first occurrence of the variable x is read as 'such that'. These expressions read as 'the set of all x such that x is a sentence of Tagalog', and 'the set of all x such that x is a student in this class whose first name starts with K '.

3.2 Set-theoretic identity. Two sets are identical iff they have exactly the same members. We symbolize identity by '='.

E.g., $\{x \mid x \text{ is a student in this class whose first name starts with } K\} = \{\text{Kali, Karen}\}$.

Because these two sets pick out the same members, we say they are the same set.

Note that:

(i) Order does not matter.

E.g., $\{\text{Blendia, Emery, Joel}\} = \{\text{Emery, Blendia, Joel}\}$.

(ii) The number of times an object is listed does not matter.

E.g., $\{\text{Blendia, Blendia, Blendia, Emery, Joel}\} = \{\text{Blendia, Emery, Joel}\}$.

Practice 1. True or false:

- | | |
|--|--|
| a. $b \in \{b, c\}$ | b. $c \in \{b, c\}$ |
| c. $\{c\} \in \{b, c\}$ | d. $\{b\} \in \{b, c\}$ |
| e. $b \in \{b, \{c\}\}$ | f. $c \in \{b, \{c\}\}$ |
| g. $\{c\} \in \{b, \{c\}\}$ | h. $\{b\} \in \{b, \{c\}\}$ |
| i. $\{b, c\} = \{c, b\}$ | j. $\{\text{Cass}\} \in \{\{\text{Cass}\}\}$ |
| k. $\text{Cass} \notin \{x \mid x \text{ is barefoot}\}$ | |

3.3 Subsets. When every member of a set A is also a member of a set B we call A a subset of B , symbolized as $A \subseteq B$.

Examples: $\{a, b\} \subseteq \{a, b, c\}$
 $\{a\} \subseteq \{a, b, c\}$
 $\{a, b, c\} \subseteq \{a, b, c\}$

Note that the null set is a subset of every set. That is, for any set A , $\emptyset \subseteq A$. The reasoning is as follows: Since \emptyset has no members, the requirement that every member of \emptyset is a member of A holds, even if vacuously. Similarly, for \emptyset to not be a subset of A , there would have to be some member of \emptyset that was not also a member of A . This is impossible since \emptyset has no members.

A is a *proper subset* of B (symbolized as $A \subset B$) iff every member of A is a member of B , and there is at least one member of B that is not a member of A .

Examples: $\{a, b\} \subset \{a, b, c\}$
 $\{a\} \subset \{a, b, c\}$
 $\{a, b, c\} \not\subset \{a, b, c\}$

Practice 2. True or false:

- a. $\{\text{Kali}\} \subseteq \{\text{Kali}, \text{Christian}, \text{Mars}\}$ b. $\{\{\text{Kali}\}\} \subseteq \{\text{Kali}, \text{Christian}, \text{Mars}\}$

Assume that $A = \{b, \{c\}\}$. True or false:

- c. $b \in A$ d. $\{b\} \subseteq A$ e. $\{c\} \in A$ f. $\{c\} \subseteq A$
g. $\{b, \{c\}\} \subseteq A$ h. $c \in A$ i. $\{\{c\}\} \subset A$ j. $\{b, \{c\}\} \not\subset A$
k. $\{b, \{c\}\} \in A$ l. $\emptyset \in A$ m. $\emptyset \in \{\emptyset\}$ n. $\emptyset \subseteq A$

3.4 Operations on sets.

The number of members in a set A is called the *cardinality of A* , written $|A|$. E.g., $|\{\text{John-Clark}, \text{Jordan}\}| = 2 = |\{a, b\}|$

The *union* of two sets A and B , written $A \cup B$, is the set whose members are just the objects that are members of A or B or both.

(5) $A \cup B =_{\text{def}} \{x \mid x \in A \text{ or } x \in B\}$

Examples: $\{a, b, c\} \cup \{a, b, \text{Andrew}\} = \{a, b, c, \text{Andrew}\}$
 $\{a, b, c\} \cup \{\{c\}, d\} = \{a, b, c, \{c\}, d\}$
 $\{\text{Andrew}\} \cup \{\text{Andrew}, \{\text{Andrew}\}\} = \{\text{Andrew}, \{\text{Andrew}\}\}$
 $\{a, b, c\} \cup \emptyset = \{a, b, c\}$

The *intersection* of two sets A and B , written $A \cap B$, is the set whose members are just the members of *both* A and B .

(6) $A \cap B =_{\text{def}} \{x \mid x \in A \text{ and } x \in B\}$

Examples: $\{a, b, c\} \cap \{a, b, \text{Andrew}\} = \{a, b\}$
 $\{a, b, c\} \cap \{\{c\}, d\} = \emptyset$
 $\{\text{Andrew}\} \cap \{\text{Andrew}, \{\text{Andrew}\}\} = \{\text{Andrew}\}$
 $\{a, b, c\} \cap \emptyset = \emptyset$

The *difference* between two sets, written $A - B$, ‘subtracts’ from A all objects in B .

(7) $A - B =_{\text{def}} \{x \mid x \in A \text{ and } x \notin B\}$

Examples: $\{a, b, \text{Cass}\} - \{\text{Cass}\} = \{a, b\}$
 $\{a, b, \text{Cass}\} - \{\text{Cass}, \{\text{Cass}\}\} = \{a, b\}$
 $\{a, b, \text{Cass}\} - \{\{b\}\} = \{a, b, \text{Cass}\}$
 $\{a, b, \text{Cass}\} - \{a, b, \text{Cass}\} = \emptyset$

The *complement* of a set A , written A' , is the set consisting of everything not in A .

(8) $A' =_{\text{def}} \{x \mid x \notin A\}$

Note: Every statement involving sets is made against a background of assumed objects, which comprise the *universe (or domain) of discourse* for that discussion, conventionally represented as U . Thus, (8) is equivalent to:

(9) $A' = U - A$

Examples: Assume that $U = \{x \mid x \text{ is in this class}\} \cup \{\text{David}\}$

$\{x \mid x \text{ is not in this class}\}' =$
 $(\emptyset)'$ =
 $(\{x \mid x \text{ is in this class}\} \cup \{\text{David}\})' =$
 $\{x \mid x\text{'s name is shorter than 9 letters}\}' =$

Ling 106, Assignment 1. Due 3 Feb 2010

Part 1. Identifying Inferences. For each of the following pairs of sentences, determine whether A entails, implicates, or is in no relation to B, and vice versa.

- (10) a. It's raining.
b. It's raining hard.
- (11) a. It's not raining.
b. It's not raining hard.
- (12) a. It's certain that he is a spy.
b. It's possible that he is a spy.
- (13) a. It's possible that he is a spy.
b. It's not certain that he is a spy.
- (14) a. Everyone is breathing.
b. Everyone is breathing deeply.
- (15) a. No one is breathing.
b. No one is breathing deeply.
- (16) a. Everyone who is breathing is happy to be alive.
b. Everyone is breathing deeply is happy to be alive.
- (17) a. I never leave early.
b. I don't always leave early.
- (18) a. I don't always leave early.
b. I sometimes leave early.
- (19) a. At most three people in this class like fruit.
b. At most three people in this class like strawberries.
- (20) a. At most three people who like fruit were at my party.
b. At most three people who like strawberries were at my party.
- (21) a. It was good.
b. It wasn't amazing.

Part 2. Set Theory.

Read PtMW, Chapter 1, and do Exercises 1, 2, 6-8.